Reg. No. : ....

Code No. : 5845 **Sub. Code : PM AM 32**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

Third Semester

Mathematics – Core

TOPOLOGY — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not a topology on $X = \{a, b, c\}$?

   (a) $\{\phi, X, \{a, b\}, \{b, c\}, \{b\}\}$

   (b) $\{\phi, X, \{a, b\}, \{b, c\}, \{b\}, \{c\}\}$

   (c) $\{\phi, X, \{a, b\}, \{b, c\}\}$

   (d) $\{\phi, X, \{a, b\}\}$

2. Which one of the following is not true?

(a) If $A = \{(\frac{1}{n})/n\epsilon Z+\}$ then $\bar{A} = \{0\} \cup A$

(b) If $B = \{0\} \cup (1,2)$ then $\bar{B} = \{0\} \cup [1,2]$

(c) If $C = Q$ then $\bar{C} = \mathbb{R}$

(d) If $D = Z_+$ then $\overline{D} = Z_+ \cup \{0\}$

3. Let π_1 be the projection of $X \times Y$ onto X. If U is open in X then $\pi_1^{-1}(U)$ is

(a) U (b) $U \times Y$

(c) $X \times U$ (d) $U \times U$

4. If $f: X \to Y$ is continuous and A is a subset of X then

(a) $f(\bar{A}) = \overline{f(A)}$

(b) $\overline{f(A)} \subset f(\bar{A})$

(c) $f(\bar{A}) \subseteq \overline{f(A)}$

(d) $\overline{f(A)} \subseteq f(A)$

5. With respect to the standard metric on $\mathbb{R}$, if $(5,9) = B(x, \Sigma)$ then

(a) $x = 5, \Sigma = 9$

(b) $x = 7, \Sigma = 2$

(c) $x = 14, \Sigma = 2$

(d) $x = 7, \Sigma = 4$

6. If d is the euclidean metric and ρ is the square metric on $\mathbb{R}^n$ then

(a) $\rho(x,y) \leq d(x,y) \leq \sqrt{2}\,\rho(x,y)$

(b) $d(x,y) \leq \rho(x,y) \leq \sqrt{n}\,d(x,n)$

(c) $\rho(x,y) \leq d(x,y) \leq \sqrt{n}\,\rho(x,y)$

(d) $\sqrt{n}\,\rho(x,y) \leq d(x,y) \leq \rho(x,y)$

7. Let $X = \{a,b,c\}, \tau = \big\{\phi, x, \{a,b\}, \{b,c\}, \{b\}\big\}$. Then

(a) $\{a\} \cup \{b,c\}$ is a separation of X

(b) $\{a,b\} \cup \{b,c\}$ is a separation of X

(c) $\{a,b,c\} \cup \phi$ is a separation of X

(d) X has no separation

8. Which one of the following set is compact in $\mathbb{R}$?

(a) $\mathbb{R}$

(b) $[0,1]$

(c) $\{0\} \cup \{(\frac{1}{n})/(n\epsilon Z+)\}$

(d) Q

9. The one point compactification of the real line $\mathbb{R}$ is isomorphic with

   (a) The sphere S^2

   (b) $\mathbb{R}$

   (c) The circle

   (d) The open interval (0,1)

10. Which one of the following is not locally compact?

    (a) Every simply ordered set having the l.u.b. property

    (b) The space R^n

    (c) The subspace Q of rational number

    (d) The real line $\mathbb{R}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Let X be a set; let Z_f be the collection of all subsets U of X such that $X - U$ either is finite or is all of X. Prove that Z_f is a topology on X.

Or

(b) Define a Hausdorff space. If X is a Hausdorff space, Prove that a sequence of points of X converges to at most one point of X.

12. (a) State and prove any two rules for constructing continuous functions.

Or

(b) Let $\{X_\propto\}$ be an indexed family of spaces; let $A_\propto \subset X_\propto$ for each $\propto$. If $\pi X_\propto$ is given either the product or the box topology, prove that $\pi \overline{A} \propto = \overline{\pi A_\propto}$

13. (a) Let d and d' be two metrics on X; Let τ and τ' be the topologies they induce, respectively. Prove that τ' is finer than τ if and only if for each x in X and each $\Sigma > 0$, there exists a $\delta > 0$ such that $B_d{}', (x, \delta) \subseteq B_d(x, \Sigma)$

Or

(b) Let $f: x \to y$; let x and y be matrizable metrics d_x and d_y respectively. Prove that the continuity of f is equivalent to the requirement that given $x \epsilon X$ and given $\Sigma > 0$, there exists $\delta > 0$ such that $d_x(x, y) < \delta \Rightarrow d_y\big(f(x), f(y)\big) < \Sigma$.

14. (a) Let $\{A_n\}$ be a sequence of connected subspaces of X, such that $A_n \cap A_{n+1} \neq \phi$ for all n. Show that UA_n is connected.

Or

(b) Show that every closed subspace of a compact space is compact.

15. (a) Show that compactness implies limit point compactness.

Or

(b) Define a locally compact space. Show that $\mathbb{R}$ is locally compact and $\mathbb{R}^{w}$ is not locally compact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the standard topology, lower limit topology and $k-$ topology on the real line $\mathbb{R}$ and obtain the relation between these topologies.

Or

(b) For a subset A of a topological space X, define the sets A' and $\bar{A}$ and show that $\bar{A} = A \cup A'$.

17. (a) Let $f: x \to y$, where x and y are topological spaces. Prove that f is continuous $\Leftrightarrow$ for every subset A of X, $f(\bar{A}) \subseteq \overline{f(A)}$ $\Leftrightarrow$ for every closed set B of y, the set $f^{-1}(B)$ is closed in X.

Or

(b) Let $f : A \to \underset{\alpha}{\pi} X_\alpha$ be given by the equation $f(a) = \left(f_\propto(a)\right)_{\propto\epsilon j}$, where $f_\alpha : A \to x_\alpha$ for each $\propto$. Let $\pi X_\propto$ have the product topology. Prove that f is continuous $\Leftrightarrow$ each function $f_\propto$ is continuous.

18. (a) Prove that the topologies on R^n induced by the Euclidean metric d and the square metric f are the same as the product topology on R^n.

Or

(b) Show that R^w in the box topology is not metrizable.

19. (a) If the sets C and D form a separation of X and if Y is a connected subspace of X, prove that Y lies entirely within either C or D and hence show that the union of a collection of connected subspaces of X that have a common point is connected.

Or

(b) Prove that the product of finitely many compact spaces is compact.

20. (a) If X is metrizable, prove that every sequentially compact space is compact.

Or

(b) If X is a locally compact Hausdorff space that is not itself compact, Prove that X has a one-point compactification.